Fundamentals of

Reinforced Concrete Design

A lecture prepared by Ernest McCullough, Chief Engineer, Fireproof Construction Bureau, Portland Cement Association, for the Short Course for Manual Training and Vocational Teachers, held at Lewis Institute, Chicago, June 26 to July 1, 1916

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Thomas A. Edison says:

"The millions of dollars of fire losses in this country annually make it a matter of moment that the superiority of reinforced concrete for fireproof structures should be thoroughly understood"

FUNDAMENTALS
of
REINFORCED CONCRETE DESIGN

Reinforced concrete is a combination of concrete and metal, preferably steel, with the two materials so disposed as regards position and *amounts* that each resists the stresses it is best fitted to resist. In piers, posts and columns the concrete takes compression assisted by the steel, and the vertical steel takes tension if any bending occurs. In beams three stresses act; namely, compression, tension and shear. The concrete takes all the compression and a limited amount of shear. The steel is computed as taking all the direct tension and assists the concrete to carry shear.

To fully understand the principles of reinforced concrete it is best to first consider materials uniform in composition, such as wood, iron or steel. Take for example a square wooden post having a length less than 15 times the thickness. This length is chosen because after the length exceeds 15 times the thickness bending can occur under heavy loading. The ratio of length to thickness (the slenderness ratio) then becomes a factor in the rules and formulas.

The column chosen is 8 x 8 in. The length is immaterial so long as we have it less than $8\times 15\div 12=10$ feet. The cross-sectional area is $8\times 8=64$ sq. in. The allowable unit load or compressive stress for the wood considered is 800 lb. per sq. in. The column can safely sustain a load of $8\times 8\times 800=51{,}200$ lb.

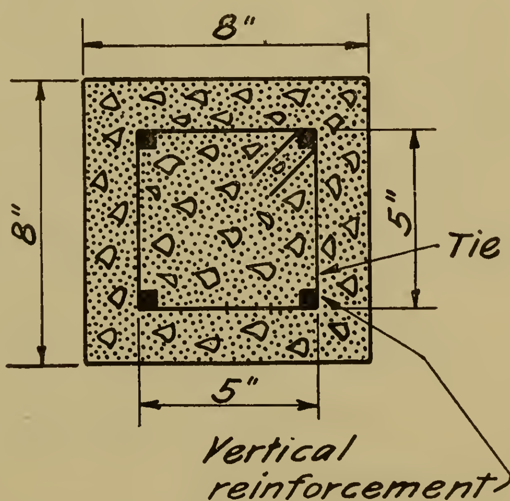

Fig. 1

Let us now consider a reinforced concrete column. For the benefit to be obtained by comparison we will make it also 8 x 8 in. outside dimensions. The steel will be in the form of four bars each ½ in. square.

The bars are set in the corners, as shown in Fig. 1, and 1½ in. in from the sides of the column. This is necessary for fire protection. The concrete outside of the steel (used for fire protection) assists in carrying

the load until the fire comes but after a severe fire it should not be depended on so we neglect it entirely in our computations. The actual area of the column is therefore $5 \times 5 = 25$ sq. in. Four ½ in. square bars have an area of one square inch. The ratio of steel to concrete is $1 \div 25 = 0.04$. The steel ratio multiplied by 100 is the per cent of steel reinforcement. Our column therefore contains 4 per cent of steel, the maximum for a column of this type.

Some building ordinances limit it to 3 per cent, following the lead of Chicago.

When a load is applied to the top of a column and the steel bars get their share they may bend because the slenderness ratio is large. It is necessary to put ties around the upright bars and these ties are spaced at intervals not exceeding 12 times the thickness of the vertical bars. Therefore, in the column under consideration, the ties will be spaced 6 inches apart because the bars are only ½ inch square. The ties are held in place by No. 18 black stove wire and the ends are turned in far enough to be gripped by the concrete so they cannot be pulled out when stressed. Ties are usually made of heavy wire or ¼ or ⅜-inch round steel rods.

Having arranged the bars and the ties, how much will our column carry?

Let us assume a 1 : 2 : 4 concrete with an allowable fiber stress in compression of 400 lb. per sq. in. The area of the concrete is 25 sq. in. less 1 sq. in. of steel = 24 sq. in., which at 400 lb. gives $24 \times 400 = 9{,}600$ lb. To determine the strength added by the steel we must be governed by the ratio of deformation between the steel and concrete. This, for the concrete we are using, is 15, as determined by experiments.

Assume a piece of steel fastened in a vertical position and a load placed on top. Assume a piece of concrete of the same size similarly placed and loaded with an equal load. Careful measurements will show that both materials shorten under the applied loads but the decrease in the length of the steel is 1/15 that of the concrete. To produce equal shortening (deformation) under equal loads the cross-sectional area of the concrete must be 15 times the cross sectional area of the steel. Thus each square inch of steel is equal to 15 square inches of concrete.

Now apply this to the column in question. The area of concrete is 24 square inches. The area of the steel is 1 square inch, the equivalent of 15 square inches of concrete. Consider the area of concrete to be increased, making it $24 + 15 = 39$ sq. in. The load-carrying capacity of the column is now $39 \times 400 = 15{,}600$ pounds. The average stress is $\frac{15{,}600}{25} = 624$ lb. per sq. in., an increase of 56 per cent. The unit stress on the steel is $15 \times 400 = 6{,}000$ lb. per sq. in.

A safe compressive stress for the steel alone would be 12,000 pounds per square inch, which shows that it is not economical to use steel in compression in reinforced concrete, except in columns.

We cannot use a steel stress exceeding the concrete stress multiplied by the ratio of deformation or the concrete will be stripped from the steel and the column will fail. The two materials must act together and shorten equally, each carrying a proportion of the load. The ratio of deformation is therefore a stress ratio for columns or for members acting wholly in compression.

The following formulas are used for the design of columns in which the unsupported length does not exceed 15 times the effective diameter or thickness; that is, the thickness of the column after deducting the protective covering of the steel.

Let f = average unit stress per sq. in. of effective area.
f_c = allowable unit stress per sq. in. on plain concrete.
p = ratio of steel to concrete.
A_c = area of concrete in sq. in.
A_s = area of steel in sq. in.
A = total effective area $= A_c + A_s$.
n = ratio of deformation.
P = total load.
then $P = Af = f_c\ (A_c + nA_s)$.
or $f = f_c\ [(1-p)+np]$.

THE HOOPED COLUMN

Make a cylinder of thin paper and fill it with sand. The paper is barely strong enough to hold the sand and if a load is put on top the paper will burst and the sand will flow. Use a tin cylinder and the pressure required to burst it will be very great. Instead of sand use cement mortar or concrete and the metal casing can be made very thin, so thin, in fact, that a wire wound spirally around the concrete cylinder will furnish the necessary strength provided the amount of metal in the wire is equal to the amount found to be necessary in the solid thin shell. Poorly made concrete needs more reinforcement than first-class concrete.

The hooped column consists of a concrete core reinforced with vertical steel and having a steel spiral around the core. There should be not less than eight vertical rods not exceeding 6 per cent of the area. The spiral hooping should be not less than one-half of one per cent and not to exceed one and one-half per cent of the area. More than this amount is wasteful, for it adds little strength. The spiral does not act until the concrete begins to fail and as it postpones the total failure the effect is the same as increasing the strength of the concrete in compression so we can use 20 to 25 per cent higher unit stress, depending upon the building ordinance followed. Steel in the form of a spiral, provided it has a pitch not exceeding one sixth of the diameter, is 2.4 times as effective as the same amount placed vertically. The vertical equivalent of spiral steel is found as follows:

Let c = circumference of the core in inches.
x = pitch of spiral in inches.
a = cross sectional area of steel used for spiral.
A = Area of core in sq. in.

Then the equivalent ratio of spiral per foot of length $= 1.1 \left\{ \dfrac{ca}{Ax} \right\}$

The strength of the hooped column is

$$P = f_c\ (A_c + nA_s + 2.4nA_h)$$

in which A_h = area of spiral steel, in terms of vertical steel.

$$\text{or } f = \frac{P}{A} = f_c\ [(1-p)+np+2.4np']$$

in which p' = ratio spiral steel expressed as equivalent vertical steel.

BEAMS

In Fig. 2 is shown a beam bending under load. In the middle of the span is shown a vertical line $A\ c$, an extension of a radial line. On one side of this line is a radial intercept $A'b$ and on the other side a radial intercept $A''d$.

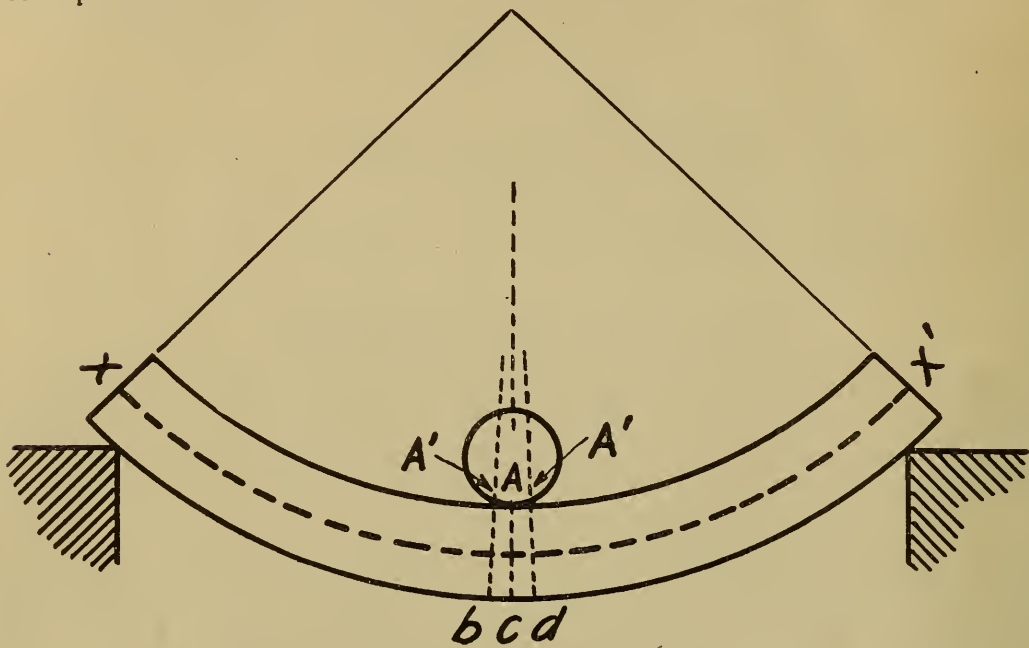

Fig. 2

Provided the material is homogeneous, that is, uniform in quality and strength, and is not stressed beyond the elastic limit, a vertical section plane before the beam bends is plane after it bends. That is, $A\ c$ is straight before the load is applied and the lines $A'b$ and $A''d$ are also straight although the horizontal separation bc is greater than $A\ A'$ and cd is greater than $A\ A''$. In Fig. 3 the line $A'b$ is assumed to be moved across Ac so the space $A\ A' = b'c$. This is equivalent to revolving the line Ac until it becomes $A'\ b'$, parallel to $A'b$.

In Fig. 4 this is again shown to illustrate the two force triangles, the upper one representing compression and the lower one tension. The

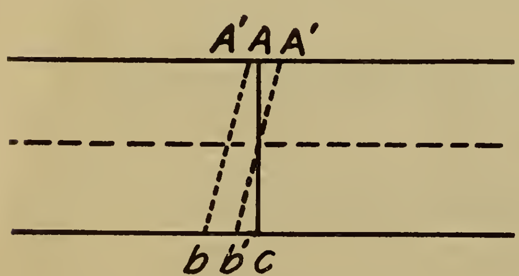

Fig. 3

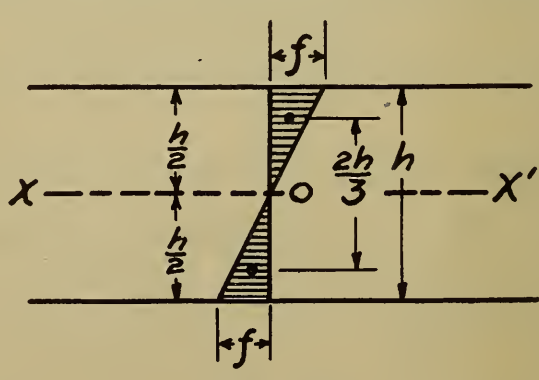

Fig. 4

material being homogeneous the neutral axis $x \ldots\ldots\ldots x'$ is midway between the top and bottom edges. The force triangles are therefore equal, the stress being zero at the neutral axis and a maximum at the edges. The maximum unit stress (skin stress some men call it) is designated by the letter f. The average stress is $\frac{f}{2}$. The area of each force triangle is

$$\frac{f}{2} \times \frac{h}{2} = \frac{fh}{4}.$$

We have been considering a thin slice of a beam, and as a beam has breadth we will use the letter b (breadth) to designate this. Our force triangles now become wedges each with a volume $= \frac{fhb}{4}$

Forces act through the center of gravity of bodies and the center of gravity of a triangle is $\frac{h}{3}$ from the base. The distance between the center of gravity of the two force triangles is $\frac{2h}{3}$ as shown in Fig. 4.

The total compressive force is equal to the total tensile force exerted to resist bending and each force wedge acts with a moment arm $= \frac{2h}{3}$, so we obtain the moment of resistance by multiplication, thus

$$M = \frac{2h}{3} \times \frac{fbh}{4} = \frac{2fbh^2}{12} = \frac{fbh^2}{6}$$ for a rectangular beam of homogeneous material; that is, one in which (below the elastic limit) the tensile strength equals the compressive strength.

In Fig. 5 is shown, a beam made of two pieces with a hinged joint. In the top of the joint is a block of rubber and at the bottom is a coiled spring. When a load is placed on top the beam will, of course, bend at

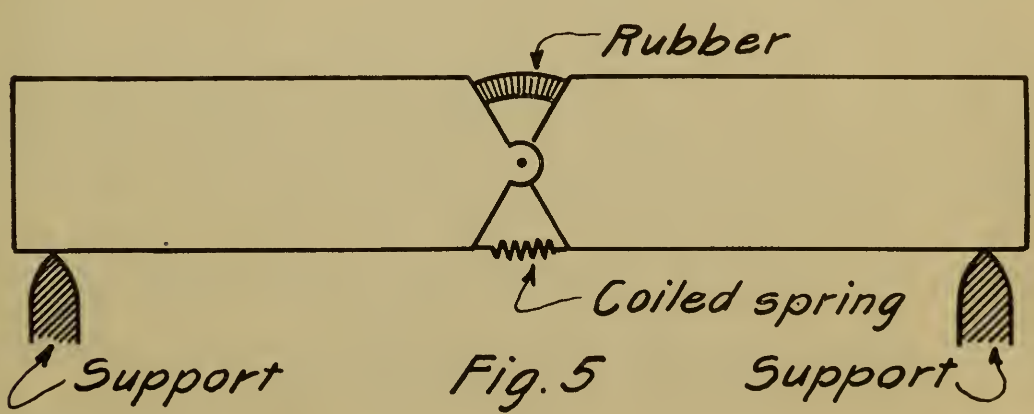

Fig. 5

the hinged joint. It requires no effort of imagination to prove that in the top of the open joint the tendency to close is opposed by the rubber and in the bottom the tendency to open is opposed by the spring. Actually

the hinge midway is not required. It merely locates definitely the position of the neutral axis, and to consider the hinge as a necessary feature is likely to confuse one as to the action of resisting forces in a beam. The neutral axis is the point where the character of stress changes from tension to compression, or from compression to tension. In a beam of homogeneous material, that is, one in which the tensile and compressive strengths are equal, with symmetrical cross-section, the neutral axis will be midway between the top and bottom surface, or skin. At the skin the stress is a maximum. At the neutral axis it is zero. A diagram illustrating this is triangular and is termed a force triangle.

In Fig. 6 (*a*) the compressive triangle has vertical lines and the tensile triangle has horizontal lines. Each triangle overlaps the other and the heavily shaded diamond center indicates a cancellation of one force by another.

The remaining effective stresses are shown in Fig. 6 (b). The neutral axis is therefore the point where the tensile and compressive stresses are definitely separated. In a beam with a finite breadth, for we have been considering only a thin vertical slice, the neutral axis becomes the neutral plane. The use of the word neutral implies a point, place or plane where there is a definite neutralization of opposite forces, or stresses. The force acting along the neutral plane is therefore horizontal shear, for the forces acting on either side are opposite in character and equal in magnitude.

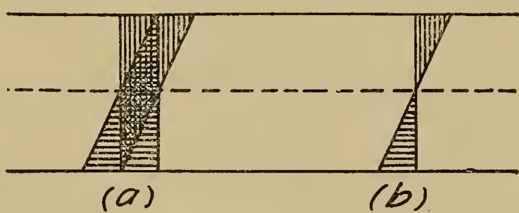

Fig. 6

The principle of the lever is evident. The length of the lever arm is the distance between centers of gravity of the opposite forces and the fulcrum is situated in the neutral plane.

In a reinforced concrete beam steel is placed near the lower edge to take all the tension, for, roughly speaking, concrete is ten times stronger in compression than in tension. In a plain concrete beam the neutral plane will be very high because the tensile and compressive forces must be equal. The tensile stress will be low and the compressive stress will be high, but the lever arm is a constant. The relative volumes of the two force wedges will be approximately as 10 is to 1.

When steel reinforcement is used the area required is computed on the assumption that it will carry all the tension and the value of the concrete in tension below the neutral axis is neglected. The tensile stress therefore does not vary from zero at the neutral axis to a maximum, but the compressive stress above the neutral axis does so vary.

The ratio of deformation between concrete and steel prevents the consideration of the value of concrete in tension. Experiments have shown that when steel embedded in concrete is stressed in tension to an amount practically equal to the tensile strength of an area of concrete equal to the steel area, multiplied by the ratio of deformation, the concrete

cracks. The tensile strength per sq. in. in the concrete at the level of the steel $= \frac{f_s}{n}$, in which f_s = unit tensile steel stress.

These cracks are vertical and fairly uniformly spaced. They probably extend as far into the beam as the neutral axis when the beam is on the point of failure. If the beam is well made and the bond of the concrete to the steel is good the cracks are so small, because numerous, that there is no danger of the entrance of moisture in large enough amounts to cause rusting of the steel. It is therefore possible to use steel with a very high stress, for the ratio of deformation is not a stress ratio as in the case of columns carrying direct compressive stress. In a column a high compressive stress on the concrete may strip it from the steel. In beams a high tensile stress in the concrete merely cracks it and the concrete between the cracks clings to the steel and protects it from corrosion. The vertical tension cracks in beams are shown in Fig. 7.

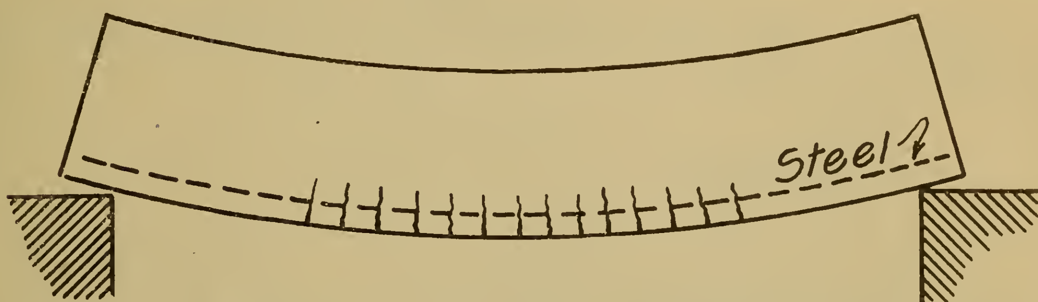

Fig. 7

The ratio of deformation plays an important part in determining the location of the neutral axis in concrete beams. In the Chicago Building Code the following values are used as a result of experiments:

Mixture	Ultimate Compressive Strength Per Square Inch	Ratio of Deformation
1 : 1 : 2	2,900	10
1 : 1½ : 3	2,400	12
1 : 2 : 4	2,000	15
1 : 2½ : 5	1,750	18
1 : 3 : 7	1,500	20

The allowable safe unit stress per sq. in. is thirty-five hundredths of the ultimate strength in compression.

In reinforced concrete we have two materials with widely differing unit stresses. The letter f is used to denote the unit stress per square inch, usually termed the "Fiber Stress." The unit steel stress is f_s and the unit concrete stress is f_c. The stress ratio $\frac{f_s}{f_c}$ is denoted by the letter m (meaning "measure"). The ratio of deformation is denoted by the letter n (meaning "number"), for it is an arbitrary number which is approximately correct.

Fig. 8 is a graphical representation of the effect n and m have on the location of the neutral plane in a reinforced concrete beam. On a piece of quadrille ruled paper plot the depth from the top of the beam to the

center of gravity of the steel by setting off ten divisions. Set off on the same scale the ratio of stresses (m) and the ratio of deformation (n) as shown. The depth to the neutral axis, k, may then be scaled. The exact value is $k=\dfrac{n}{n+m}$.

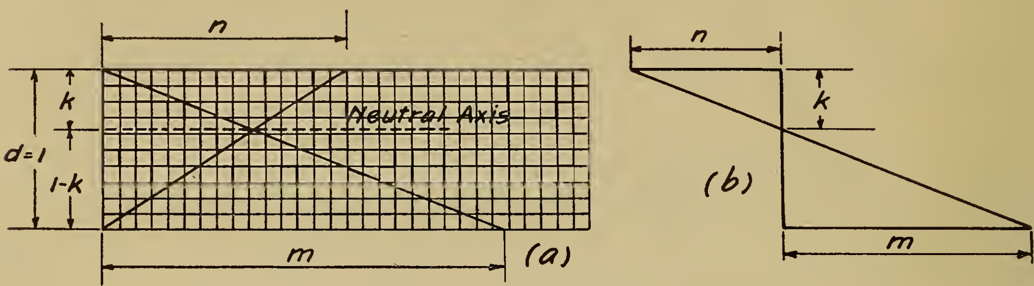

Fig. 8

EXAMPLE. What is the value of k when $f_s=16{,}000$ lb. per sq. in. and $fc=650$ lb. per sq. in.?

$$n=15$$

$$m=\frac{16000}{650}=24.62$$

$$k=\frac{n}{n+m}=\frac{15}{15+24.62}=0.378\ d$$

Fig. 9 shows the force triangle of the concrete in compression and the steel in tension. To find the ratio of steel, proceed as follows:

The total amount of compressive force is found by obtaining the area of the force triangle. The height is kd and the average stress is

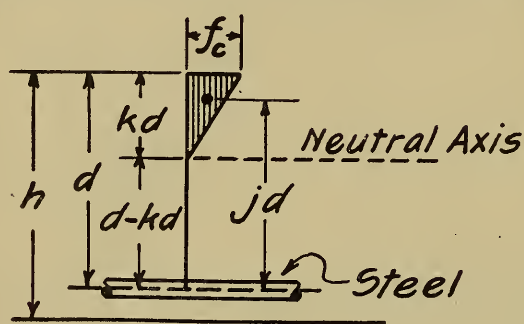

Fig. 9

$\dfrac{f_c}{2}$. The compression $=\dfrac{kdf_c}{2}$. We use kd for the depth, d, has a definite value and k is a percentage of d. The moment arm, the lever, has a length $=d-\dfrac{kd}{3}$ or jd. ($j=1-\dfrac{k}{3}$). The total compressive force in a thin slice, $C=\dfrac{f_c}{2}\,kdjd=\dfrac{f_c}{2}kjd^2$

The tensile force $T = f_s A j d$, in which A = area of steel in sq. in. The steel ratio $= \frac{A}{bd} = p$, so it is necessary to introduce the breadth b, into the expression.

We now have

$$C = \frac{f_c}{2} kjbd^2$$

and

$$T = f_s p j b d^2$$

then

$\frac{f_c k}{2} = f_s p$, provided $C = T$, which is the case for a beam with "balanced" reinforcement.

Assigning values, $\frac{650}{2} \times 0.378 = 16000p$

and

$$p = \frac{325 \times .378}{16000} = 0.00767$$

$$p \times 100 = \text{per cent of steel} = 0.767\ (0.77\%).$$

By formula $p = \frac{k}{2m}$

Assume a beam 6 inches wide with depth to the center of the steel = 9 ins. What is the resisting moment?

$C = 325 \times 0.378 \times 0.874 \times 6 \times 9^2 = 52{,}242$ in lb.

What area of steel will be required?

$A = bdp = 6 \times 9 \times 0.0077 = 0.42$ sq. in.

Check the steel:

$T = f_s A j d = 16000 \times 0.42 \times 0.874 \times 9 = 52{,}859$ in lb.

The greater resisting moment in tension is due to having used 0.42 sq. in. of steel, the exact area being 0.4158 sq. in. The area of steel used is governed by the commercial sizes of bars and rods, or the expanded metal or wire fabric used. The actual steel area used will usually be greater than the theoretical area necessary.

Every reinforced concrete beam has two moments of resistance; one determined by the concrete, the other by the steel. The lesser of the two is the resisting moment which determines the actual strength. In designing slabs a width of one foot is taken, for a slab is merely a wide and shallow beam assumed to be made up of a number of beams each 12 inches wide.

R is a moment factor. For the concrete $R = \frac{f_c kj}{2}$. For the steel $R = f_s pj$. Then the bending moment $M = Rbd^2$

$$b = \frac{M}{Rd^2}$$

$$d = \sqrt{\frac{M}{Rb}}$$

To design a beam select stresses for the steel and concrete and find R. The value of M is the bending moment which is equal to, or is less than, the resisting moment. Assume a breadth and solve for the depth, or assume a depth and solve for the breadth. T beams are beams in which the floor slab is considered to be a part of the beam and carries the compression. The breadth, b, is the width in the floor slab and the stem below the slab must be wide enough to contain the reinforcement. The width necessary must include space between the bars and on each side to furnish bond and shearing strength.

Below the steel there must be a covering of concrete not less than the thickness of the steel, with a minimum thickness of one-half inch, this being for bond and fire protection. In all building ordinances minimum coverings of concrete are specified; as for example, one-half inch for slabs and one and one-half inches for beams, girders and columns.

SHEAR

Fig. 10 shows a beam with typical shear cracks. A beam may fail by crushing of the concrete in the top, by the stretching or slipping of the steel or by shear, which is manifested by the appearance of shearing

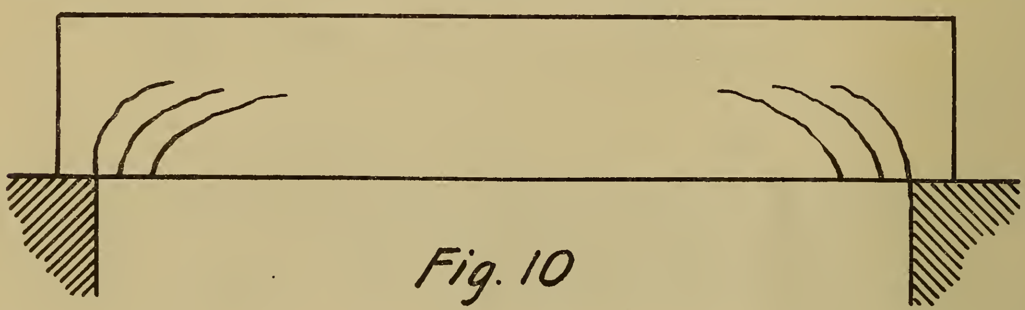

Fig. 10

cracks. These cracks are an indication of tension in the concrete and stirrups are used to prevent shearing (diagonal tension) failures.

Fig. 11 (a) shows a uniformly loaded beam resting freely on two supports. At (b) is shown the shear diagram. The vertical shear at either end is equal to one half the load and is zero at the point of maximum bending moment. The vertical depth, measured in pounds, of the shear diagram at any point is a sum-curve of the loading to that point. (See page 13.)

The bending moment at any point is the area of the shear diagram between that point and the support. The vertical dimension is in pounds. The horizontal dimension is in feet when the result is foot pounds and in inches when the result is inch pounds. The bending moment at any section, such as $y \ldots\ldots\ldots\ldots y$ is the sum curve of the shear at that section.

Let the bending moment at $y \ldots\ldots\ldots\ldots y$ be M (in inch pounds),

then the unit fiber stress at $y \ldots\ldots\ldots\ldots y = \dfrac{M}{jdb}$.

Let V = total shear at any section.

M = bending moment on one side of the section, and

M' = bending moment on the other side of the section, the section assumed to have a thickness as nearly zero as possible.

Then on one side

$$f = \frac{M}{jdb}$$

And on the other side

$$f = \frac{M'}{jdb}$$

The total shear is $V = M - M'$, and the unit shear is

$$v = \frac{V}{jdb} = \frac{M - M'}{jdb}$$

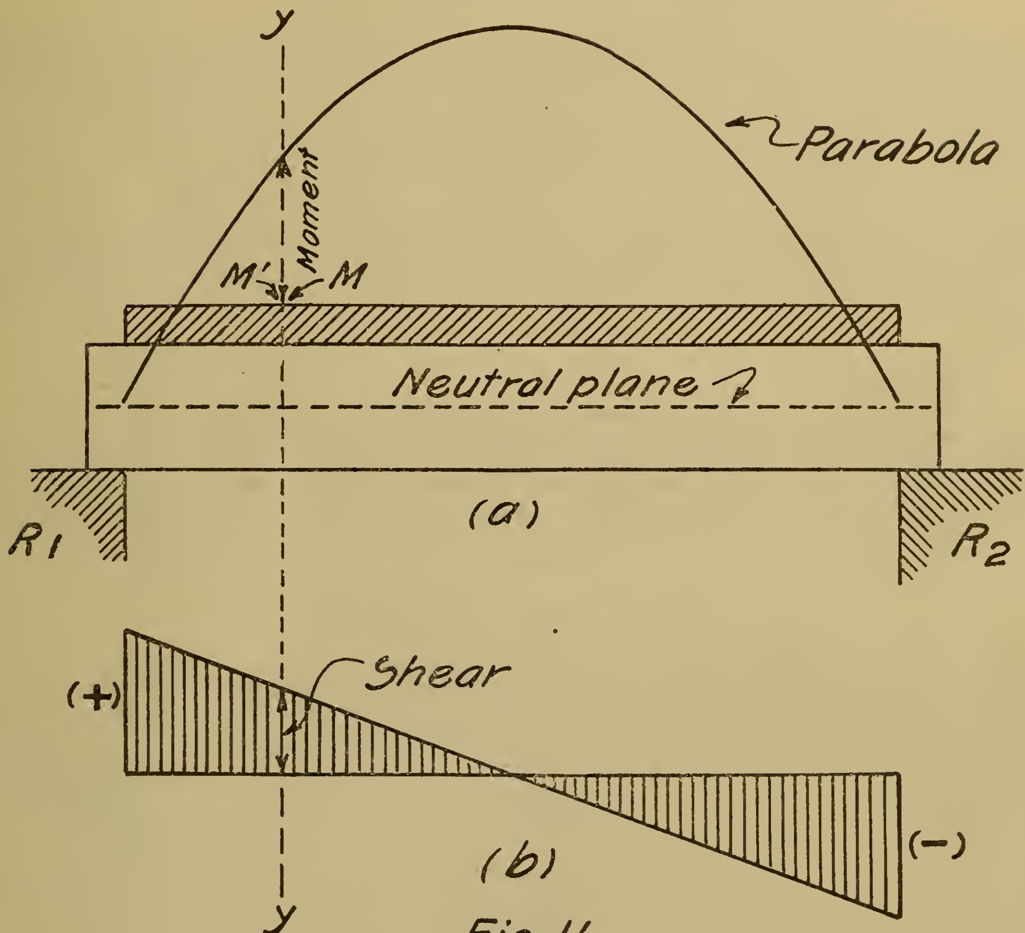

Fig. 11

In T beams the b used in expressions for shear is the thickness of the stem below the floor slab.

The shear being tension increases from the top and bottom skin to a maximum at the neutral axis where the bending stress changes from tension to compression and from compression to tension. In a beam of homogeneous material with a uniform cross section the shear is the same above and below the neutral plane at equal distances from this plane. The amount may be computed for any depth by assuming different values for jd, the V and M being constant. This distribution of horizontal shear is found in a reinforced concrete beam above the neutral plane,

Below the neutral plane the shear is constant, for all the tension is carried by the steel. The unit value of the shear at all depths from the neutral axis to the center of gravity of the steel is determined by the expression,

$$v = \frac{V}{jdb}$$

in which d = depth from top of beam to center of gravity of the steel

$$j = 1 - \frac{k}{3}$$

$$jd = d - \frac{k}{3}$$

In Fig. 12 is given an illustration of horizontal shear. Several planks laid loosely on end supports bend under load and the slipping of one plank past the adjoining plank is a horizontal movement. This is shown in Fig. 12a. Spike, or bolt, the planks together as shown in Fig. 12b and the slipping cannot occur. The spikes or bolts represent with fair accuracy the stirrups used in reinforced concrete beams. The shear being a maximum at the ends where the bending moment is a minimum, the fastenings are closer together than nearer the middle of the span where the shear is a minimum and the bending moment is a maximum.

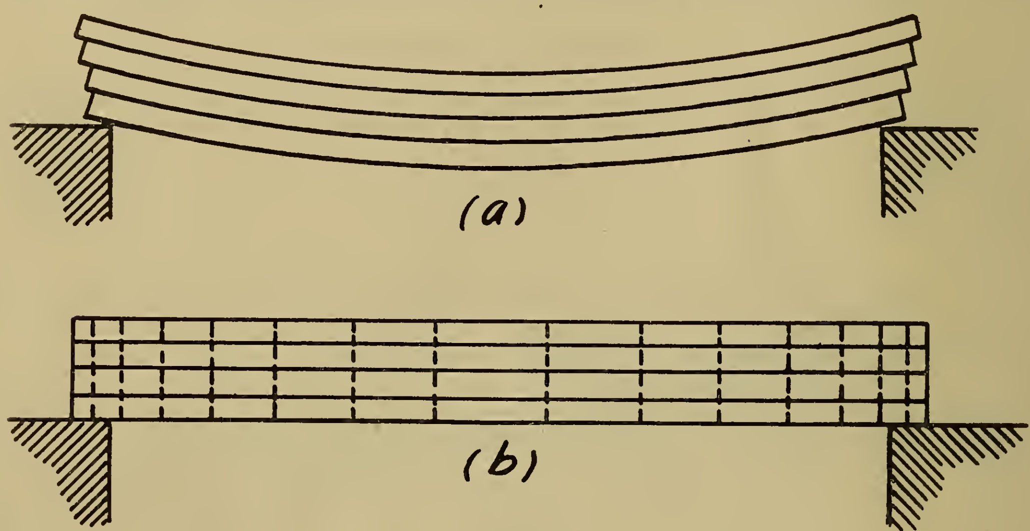

Fig. 12

By reference to Fig. 11b it is seen that vertical shear exists at all points on a beam. We have seen that horizontal shear also exists at all points on a beam. Along the neutral plane it is equal to the maximum end vertical shear. Very thin horizontal slices, like the planks in Fig. 12, are assumed for the purposes of computation.

The resultant, according to the parallelogram of forces, is a diagonal tension, which causes the cracks shown in Fig. 10.

In designing reinforced slabs, beams and girders the resisting moment must be equal to or greater than the bending moment. When this is fixed the beam must be tested for shearing strength, and if it is found to be deficient in this particular, steel in the form of stirrups must be provided

or the size of the beam be increased. The unit shearing stress v should not exceed 40 lb. per sq. in. for the concrete alone, nor exceed a total of 120 lb. per sq. in. when stirrups are used.

BOND

Experiments have shown that 70 lb. per sq. in. is a safe allowance for bond in order that the steel and concrete may act together. The coefficient of expansion of the two materials is practically the same, so we need not fear a separation under extreme variations in temperature.

Assuming the two materials to act together and the safe bond stress is 70 lb. per sq. in., what length of embedment is necessary for a bar one-inch square stressed 16,000 lb. per sq. in.?

The one-inch square bar has four square inches of surface for each inch of length.

$$\frac{16{,}000}{4 \times 70} = 57.142 \text{ in. embedment required.}$$

Four ½-inch square bars have the same area as one 1-inch square bar but the surface $= 4\ (4 \times \frac{1}{2}) = 8$ sq. in. and the length of embedment $= \dfrac{16{,}000}{8 \times 70} = 28.57$ in.

When a large bar having sufficient area to carry the tension is found to be deficient in bonding area, smaller bars may be used. The stress is the maximum tensile stress in the reinforcement at the point of maximum bending moment, and the reinforcement each side of this point must be long enough for bond.

BENDING UP STEEL

The bending moment decreases toward the supports and when a number of bars are used they may gradually be decreased in number, always allowing not less than two to go the full length in the bottom. The other bars are turned up a short distance past the point where they are no longer needed for direct tension, being carried to within an inch of the top of the beam on an angle of 45 degrees or less, and thence horizontally to the supports. Half the steel area may thus be bent up at 0.23 of the span from the support, one half the remainder at 0.15 of the span and one half of the remainder at 0.10 of the span. These rules are closely approximate and apply only to uniformly loaded beams. Exact rules are given in text books. Bent up steel assists in reinforcing the web or body of the beam and thus strengthens the beam against failure by shear.

STIRRUPS

Stirrups should be fastened to the tension steel, not merely looped around it. They should extend far enough above the neutral axis to develop bond. The stress in the stirrups is tension and it is equal to the tension in the concrete multiplied by the ratio of deformation at the instant the concrete cracks, when the whole stress is immediately taken by the steel. The stress used for stirrups should not exceed 75 per cent of the stress in the tension reinforcement.

Fig. 13 represents a unit vertical slice of a beam.

V = total vertical shear at any section.
w = load in pounds per lineal foot.
L = span of beam in feet.
b = breadth of beam in inches.
d = depth to center of steel in inches.
j = moment arm (percentage of d).
v = unit shear.
v_c = unit shear carried by concrete.
v_s = unit shear carried by steel.
l = distance in inches to point where concrete alone can carry the shear.
T' = total tension of horizontal shear due to shear carried by the steel.
D_t = diagonal tension at neutral plane.
f_s = unit tensile steel stress.
A_w = area of web steel = $A_b + S_a$.
A_b = area of bent up bars.
S_a = area of stirrup steel.

Then (for uniformly loaded beams)—

(1) $V = \frac{wL}{2}$ at supports $= \frac{wL}{2} - wx$ at any point distant, x, from support. (See Fig. 11.)

(2) $v = \frac{V}{jdb}$ (max. 120 lb.) (3) $v_s = v - v_c$ (v_c = 40 lb. per sq. in.)

(4) l (in ins.) $= \frac{6v_sL}{v}$ (5) $T' = \frac{Lv_sb}{2}$

(6) $D_t = \frac{T'}{\sqrt{2}} = 0.707T'$ (7) $A_w = \frac{D_t}{f_s}$

(8) $S_a = A_w - A_b$.

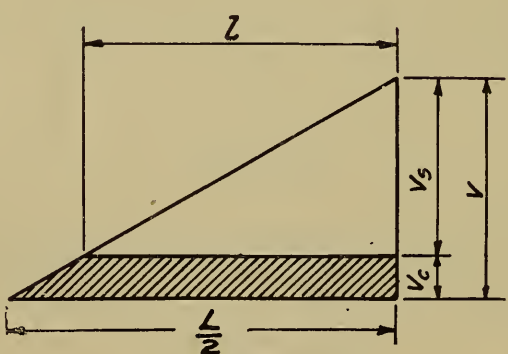

Fig. 13

The stirrups should be equal in area and the intervals should increase by increments of 50% or logarithmically, the greatest interval being equal to d. The area of steel in the bent up bars should be neglected when it is more than 0.15L from the supports, for the maximum shear is at the edge of the support at either end. When four stirrups are used at each end, the first will be $\frac{1}{4}d$ from the support; the second $\frac{1}{2}d$ from the first; the third $\frac{3}{4}d$ from the second and the fourth d from the third.

REINFORCING A CIRCULAR TANK

In a round tank the only stress is tension, being equal to $\frac{wd}{2}$.

in which w = weight (pressure).
d = diameter.

The tension is figured for strips one foot wide, the weight used being the weight of the liquid at the depth of the strip. The weight of one cubic foot of water being 62.5 lb., w at the depth of 10-ft. is 625 lb. At 15 feet the weight is 937.5 lb.

The stress to be used in the steel cannot exceed $\frac{f_s}{n}$, or large cracks will open. The concrete shell should be designed to carry all the stress with an assumed safe unit tensile stress. Steel should be used to carry all the tensile stress. This combination fixes the unit stress in the steel as equal to nf_c until something causes the tank wall to crack, after which all the tension is carried by the steel with the assumed fiber stress, f_s.

Owing to the difficulties encountered in construction no tank wall should have a thickness of less than six inches, regardless of the theoretical thickness found by computation.

Example: Give the proper thickness of wall and amount of steel required for a circular tank 20 feet in diameter at a depth of 14 feet, using a 1 : 2 : 3 concrete.

$$T = \frac{wd}{2} = \frac{14 \times 62.5 \times 20}{2} = 8{,}750 \text{ lb.}$$

Use a tensile stress of 12,000 lb. per sq. in. in the steel in order to care for possible mistakes in connecting ends of bars. Clamps should not be used. The best method is to have the ends of the bars overlap a length of not less than 40 times the thickness. The overlapping ends should not be in contact, but should be separated to leave a space of about twice the thickness of the bars so the concrete may surround the steel.

$$A_s = \frac{8750}{12000} = 0.729 \text{ sq. in. (area of steel).}$$

Use three ½-in. sq. bars, giving an area of 0.75 sq. in.

A safe tensile stress for well made 1 : 2 : 3 concrete is 175 lb. per sq. in.

Area of concrete $= \frac{8750}{175} = 50$ sq. in. The theoretical thickness of the wall is $\frac{50}{12} = 4.166$ in., for the strip is 12-ins. wide. The thickness, for reason given, should be not less than 6-ins., so the actual area will be 72 sq. in.

The area of the concrete is 72 sq. in. minus area of steel $= 72 - .075 = 71.25$ sq. in.

The ratio of deformation for 1 : 2 : 3 concrete is 12, so the steel is equivalent to a concrete area of $12 \times 0.75 = 9$ sq. in.

Adding : $71.25 + 9 = 80.25$ sq. in.

The average stress is $8750 \div 72 = 121.53$ lb. per sq. in., and the stress on the concrete is $8750 \div 80.25 = 109.03$ lb. per sq. in.

The stress in the steel, when both materials are carrying tension, is $12 \times 109.03 = 1308.36$ lb. per sq. in.

If, for any reason, the concrete cracks the steel will carry all the tension with a stress of 12,000 lb. per sq. in. Cracks may occur where an occasional poorly mixed batch of concrete was deposited; where construction joints are defective; where forms were removed too roughly; through ice pressure; by reason of excessive temperature changes; frequent and sudden changes in pressure due to quick filling or emptying of water; through defective foundations. When the steel stress is not permitted to exceed 12,000 lb. per sq. in. the only effect of cracking will be to throw the entire tension on the steel and the cracks will not be large enough to cause corrosion or leakage.

LITERATURE

This lecture is not intended to completely cover the subject of reinforced concrete design. It is merely an introduction to the subject. The following books are recommended as texts:

"Principles of Reinforced Concrete Construction," by Turneaure & Maurer, $3.50, published by John Wiley & Sons, New York, N. Y.

"Concrete, Plain and Reinforced," by Taylor & Thompson, $5.00, published by John Wiley & Sons, New York, N. Y.

Three books by Prof. Geo. A. Hool, intended for self-tutored men; entitled "Reinforced Concrete Construction," published by McGraw-Hill Publishing Company, New York, N. Y.

Vol. 1 Fundamental Principles, $2.50.
Vol. 2 Retaining Walls and Buildings, $5.
Vol. 3 Bridges and Culverts, $5.

For data on practical design and construction, "Recommended Building Code," prepared by the National Board of Fire Underwriters, 76 William Street, New York, N. Y. This building code should be used in all technical schools as a text book on modern building construction.

Tennis Courts of Concrete
Concrete Septic Tanks
Concrete Fence Posts
Small Concrete Garages
Concrete Feeding Floors, Barnyard Pavements and Concrete Walks
Specifications for Concrete Roads, Streets and Alleys
Specifications for Concrete Pavement Between Street Car Tracks
Facts Everyone Should Know About Concrete Roads
Concrete Facts About Concrete Roads
Proportioning Concrete Mixtures and Mixing and Placing Concrete
Concrete Foundations
Concrete Troughs, Tanks, Hog Wallows, Manure Pits and Cisterns
Concrete Sewers
Concrete Swimming and Wading Pools and How to Build Them
Concrete Linings for Irrigation Canals
Concreting in Cold Weather
That Alley of Yours
Concrete Houses and Why to Build Them

Are booklets of distinct educational value, free from advertising, and in most cases will serve as supplemental classroom texts. If interested in any of these publications, write the

Portland Cement Association
111 West Washington Street
CHICAGO

"Concrete for Permanence"